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Numerical Analysis of Trajectories of a Quantum Particle in Two-Slit Experiment

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It is well known that the concept of a trajectory of a quantum particle is itself nonsense in the so-called "Copenhagen" interpretation. However, if the interpretation proposed by Ishikawa [*International Journal of Theoretical Physics*, **30**(4), 401 (1991)] can be accepted in quantum mechanics, the trajectory of a quantum particle is significant (though it includes errors). In this paper we numerically analyze discrete trajectories of a quantum particle in a two-slit experiment under this new interpretation.

1. INTRODUCTION

It is well known that the concept of a trajectory of a quantum particle is nonsense in the so-called "Copenhagen" interpretation. However, if the interpretation proposed by Ishikawa (1991*b*) can be accepted in quantum mechanics, the (discrete) trajectory of a quantum particle is significant enough (though of course it includes errors). Now we briefly explain this new interpretation, which gives the foundation to analyze a discrete trajectory of a quantum particle in numerical analysis.

Let V be a Hilbert space with the inner product $\langle \cdot, \cdot \rangle_V$. $(X, \mathscr{F}, F)$ is called a projection-valued probability space if it is provided with a projection-valued probability F on a complete separable metric space X (with a Borel field $\mathscr{F}$) in a Hilbert space V such that:

(a) For every $\Xi \in \mathscr{F}$, $F(\Xi)$ is a projection in V such that $F(\emptyset) = 0$ and $F(X) = I$, where 0 is a 0-operator and I is an identity operator in V.

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(b) For any countable decomposition $\{\Xi_j\}_{j=1}^{\infty}$ of Ξ (Ξ_j, $\Xi \in \mathscr{F}$), $F(\Xi) = \sum_{j=1}^{\infty} F(\Xi_j)$, holds where the series is weakly convergent.

Projection-valued probability space was firstly introduced in quantum mechanics by Davies (1976), who chiefly investigated positive-operator-valued probability space as a generalization of projection-valued space. In this paper, we shall use the projection-valued probability spaces as the mathematical model of observables. So a projection-valued probability space $(X, \mathscr{F}, F)$ is also called an observable in V in this paper. Note that any self-adjoint operator A in V has a spectral representation $A = \int_{\mathbf{R}} \lambda\, E_A(d\lambda)$ (see, e.g., von Neumann, 1932, and Prugovečki, 1981). So we sometimes identify the self-adjoint operator A with the observable $(\mathbf{R}, \mathscr{B}, E_A)$, where $\mathscr{B}$ is a Borel σ-field on a real field $\mathbf{R}$.

Let ψ be a (pure) state of a system S in a Hilbert space V, that is, $\psi \in V$ with $\|\psi\|_V = 1$. Let $(X, \mathscr{F}, F)$ and $(Y, \mathscr{G}, G)$ be observables in V. Put

$$\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)} = \{\Xi \in \mathscr{F} \mid G(\Gamma)F(\Xi)P_{(F,\psi)} = F(\Xi)G(\Gamma)P_{(F,\psi)} \quad (\forall \Gamma \in \mathscr{G})\}$$

where $P_{(F,\psi)} \equiv$ "the projection on the smallest closed subspace that contains $\{F(\Xi)\psi \mid \Xi \in \mathscr{F}\}$." It is clear that $\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$ is a σ-subfield of $\mathscr{F}$ such that $(X, \mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}, F)$ and $(Y, \mathscr{G}, G)$ commute with respect to ψ, that is, $G(\Gamma)F(\Xi)\psi = F(\Xi)G(\Gamma)\psi$ holds for all $\Gamma \in \mathscr{G}$ and $\Xi \in \mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$. When $(Y, \mathscr{G}, G)$ $[=(\mathbf{R}, \mathscr{B}, E_A)]$ is an observable representing a self-adjoint operator A [i.e., $A = \int_{\mathbf{R}} \lambda\, E_A(d\lambda)$], we sometimes write $\mathscr{F}^{\psi}_{A}$ instead of $\mathscr{F}^{\psi}_{(\mathbf{R},\mathscr{B},E_A)}$.

Remark 1. We have another proposal about $\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$ (Ishikawa, 1992) as follows. Let $\mathscr{A} = \{\mathscr{F}^0 \mid \mathscr{F}^0$ is σ-subfield of $\mathscr{F}$ such that $F(\Xi)G(\Gamma)\psi = G(\Gamma)F(\Xi)\psi$ $(\forall \Xi \in \mathscr{F}^0, \forall \Gamma \in \mathscr{G})\}$. It is easily shown that $\mathscr{A}$ has a maximal element. So we can also define a σ-subfield $\bar{\mathscr{F}}^{\psi}_{(Y,\mathscr{G},G)}$ as $\bar{\mathscr{F}}^{\psi}_{(Y,\mathscr{G},G)} = \bigcap\{\mathscr{F}^0 \mid \mathscr{F}^0$ is a maximal element of $\mathscr{A}\}$. Even if we use this $\bar{\mathscr{F}}^{\psi}_{(Y,\mathscr{G},G)}$ instead of $\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$, all arguments in this paper hold as for $\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$.

We define a conditional probability $\mu_\psi(x, \Gamma)$, or precisely, $\mu_\psi(x, \Gamma$: $(X, \mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}, F), (Y, \mathscr{G}, G))$ that satisfies the following conditions:

(a) For each Γ $(\in \mathscr{G})$, $\mu_\psi(x, \Gamma)$ is $\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$-measurable as a function of x and $0 \le \mu_\psi(x, \Gamma) \le 1$, and for each x $(\in X)$, $\mu_\psi(x, \cdot)$ is a probability measure on $(Y, \mathscr{G})$.

(b) For each $\mathscr{F}^{\psi}_{(Y,\mathscr{G},G)}$-measurable function f: $X \to \mathbf{R}$ and $\Gamma \in \mathscr{G}$,

$$\int_X f(x)\langle \psi, F(dx)G(\Gamma)\psi \rangle_V = \int_X f(x)\mu_\psi(x, \Gamma)\langle \psi, F(dx)\psi \rangle_V$$

The existence and uniqueness (in some sense) of $\mu_\psi(x, \Gamma)$ were assured in Ishikawa (1991*b*) and Ash (1972). So the following symbolic representation of $\mu_\psi(x, \Gamma)$ is available for our arguments in this paper:

$$\mu_\psi(x, \Gamma: (X, \mathscr{F}^\psi_{(Y,\mathscr{G},G)}, F), (Y, \mathscr{G}, G)) = \lim_{\substack{\Xi \to x \\ \mathscr{F}^\psi_{(Y,\mathscr{G},G)} \ni \Xi \ni x}} \frac{\langle \psi, F(\Xi)G(\Gamma)\psi \rangle_V}{\langle \psi, F(\Xi)\psi \rangle_V} \quad (1)$$

Axiom 2 (Born's probabilistic interpretation and generalized Copenhagen interpretation). Let ψ be a state of a system S in a Hilbert space V. Let $(X, \mathscr{F}, F)$ and $(Y, \mathscr{G}, G)$ be any observables in V. Then:

(i) The probability that x_0 $(\in X)$, the measurement value obtained by the measurement of the observable $(X, \mathscr{F}, F)$ for this system S, belongs to a set Ξ $(\in \mathscr{F})$ is given by $\langle \psi, F(\Xi)\psi \rangle_V$.

(ii) When we get x_0 $(\in X)$ by the measurement of the observable $(X, \mathscr{F}, F)$ for this system S, the probability that $\hat{y}_0$ $(\in Y)$, the "true" value of the observable $(Y, \mathscr{G}, G)$ for this system S, belongs to a set Γ $(\in \mathscr{G})$ is given by $\mu_\psi(x_0, \Gamma: (X, \mathscr{F}^\psi_{(Y,\mathscr{G},G)}, F), (Y, \mathscr{G}, G))$.

Now we have the following definition.

Definition 3 (Approximate simultaneous measurement in average sense and its average error). Let H be a Hilbert space with the inner product $\langle \cdot, \cdot \rangle_H$. Let $A_0, A_1, \ldots, A_{N-1}$ be any physical quantities (i.e., self-adjoint operators) in a Hilbert space H. A quartet $\mathbf{M} = (K, v, (X, \mathscr{F}, F), f = (f_0, \ldots, f_{N-1}))$ is called an approximate simultaneous measurement of $\{A_k\}_{k=0}^{N-1}$ in H if it satisfies the following conditions:

(i) v is an element in a Hilbert space K such that $\|v\|_K = 1$, and $(X, \mathscr{F}, F)$ is an observable in a tensor Hilbert space $H \otimes K$ and $f: X \to \mathbf{R}^N$ is a measurable map.

(ii) Put $\hat{A}_k = \int_X f_k(x) F(dx)$ $(k = 0, 1, \ldots, N-1)$; then, for each k, a set $D_v(\hat{A}_k)$ $(\equiv \{u \in H: u \otimes v \in D(\hat{A}_K)$, the domain of $\hat{A}_k\})$ is a core of A_k, i.e., A_k is essentially self-adjoint on $D_v(\hat{A}_k)$.

(iii) For each k, $\langle u, A_k u \rangle_H = \langle u \otimes v, \hat{A}_k(u \otimes v) \rangle_{H \otimes K}$ $(\forall u \in D_v(\hat{A}_k))$.

Then, $\bar{\delta}_M(A_k, u)$, the kth average error in the measurement $\mathbf{M}$ with respect to a state u $(\in H)$, is defined by

$$\bar{\delta}_{\mathbf{M}}(A_k, u) \equiv \left[\int_X \left| \int_{\mathbf{R}} |f_k(x) - \xi|^2 \mu_{u \otimes v}(x, d\xi) \right| \langle u \otimes v, F(dx)(u \otimes v) \rangle \right]^{1/2} \quad (2)$$

where $\mu_{u \otimes v}(x, d\xi)$ is defined as in Axiom 2. Also, $\{\bar{\delta}_{\mathbf{M}}(A_k, u)\}_{k=0}^{N-1}$ is called an average error in the measurement $\mathbf{M}$ with respect to a state u.

The approximate simultaneous measurement was first introduced in Davies (1976), Holevo (1982), etc. If we accept Axiom 2, the average error in the measurement $\mathbf{M}$ is naturally derived (Ishikawa, (1991*b*). We note that the existence of some approximate simultaneous measurement of $\{A_k\}_{k=0}^{N-1}$ is proved in Abu-Zeid (1987) or Ishikawa (1991*a*) in detail.

Furthermore, in the particular case $N = 1$, Ishikawa (1991*b*) has proved the following Heisenberg uncertainty relation under some conditions:

$$\bar{\delta}_{\mathbf{M}}(A_0, u) \cdot \bar{\delta}_{\mathbf{M}}(A_1, u) \geq |\langle A_1 u, A_0 u\rangle - \langle A_0 u, A_1 u\rangle|/2 \tag{3}$$

2. ANALYSIS OF TRAJECTORIES IN TWO-SLIT EXPERIMENT

In this section, we shall analyze a discrete trajectory of a quantum particle under the interpretation mentioned in Axiom 2. The main ideas have been mentioned in Ishikawa (1991*b*). We develop the ideas of Arthurs and Kelly (1965) and She and Heffner (1966).

We shall consider a particle P in the one-dimensional real line $\mathbf{R}$, whose initial state function is $u(x) \in H \equiv L^2(\mathbf{R})$. Since our purpose is to analyze the discrete trajectory of the particle in the two-slit experiment, we choose the state $u(x)$ as follows:

$$\begin{aligned} u(x) &= 1/\sqrt{2}, \qquad x \in (-3/2, -1/2) \cup (1/2, 3/2) \\ &= 0, \qquad \text{otherwise} \end{aligned} \tag{4}$$

Let A be a position observable in H, that is, $(Av)(x) = xv(x)$ for $v(x) \in H \equiv L^2(\mathbf{R})$. We treat the following Heisenberg kinetic equation of the time evolution of the observable A_t $(-\infty < t < \infty)$ in a Hilbert space H with a Hamiltonian $\mathscr{H} = -(\hbar^2/2m)\, \partial^2/\partial x^2$:

$$-i\hbar \frac{dA_t}{dt} = \mathscr{H} A_t - A_t \mathscr{H}, \qquad -\infty < t < \infty, \quad \text{where } A_0 = A \tag{5}$$

Put $\theta > 0$ and $N \geq 2$ (integer). Now we consider the approximate simultaneous measurement $\mathbf{M}$ of self-adjoint operators $\{A_{\theta k}\}_{k=0}^{N-1}$ for a particle P with an initial state $u(x)$. An easy calculation shows that

$$A_t = U_{-t} A U_t = U_{-t} x U_t = x + \frac{\hbar t}{im} \frac{d}{dx} \tag{6}$$

where the one-parameter unitary group U_t is defined by $\exp(-i\hbar^{-1}\mathscr{H} t)$. Let

$$V = H \otimes K = H \otimes \left(\bigotimes_{k=1}^{N-1} H \right) = \bigotimes_{k=0}^{N-1} H = L^2(\mathbf{R}^N) \qquad \text{and} \qquad \hat{U}_t = \bigotimes_{k=0}^{N-1} U_t$$

Let a_{kn} $(k, n = 0, 1, \ldots, N-1)$ be real numbers such that $\sum_{n=0}^{N-1} \alpha_{kn}\alpha_{ln} = 0$ $(k \neq l)$ and $\alpha_{k0} = 1$ $(\forall k)$. Define self-adjoint operators $\hat{A}_{\theta k}$ $(k = 0, 1, \ldots, N-1)$ in V $[\equiv L^2(\mathbf{R}^N)]$ by

$$\hat{A}_{\theta k} = \sum_{n=0}^{N-1} \alpha_{kn} \left(x_n + \frac{\hbar\theta k}{im} \frac{\partial}{\partial x_n} \right) \tag{7}$$

It is clear that $\hat{A}_{\theta k}$ $(k = 0, 1, 2, \ldots, N-1)$ commute. Also, for each k

($k = 0, 1, 2, \ldots, N-1$), $\hat{A}_{\theta k}$ and $A_{\theta k} \otimes I\,[\equiv x_0 + (\hbar\theta k/im)\,\partial/\partial x_0]$ commute. Let $\hat{E}_{\theta k}$ be the spectral measure of $\hat{A}_{\theta k}$ [i.e., $\hat{A}_{\theta k} = \int_{\mathbf{R}} \lambda \hat{E}_{\theta k}(d\lambda)$]. From the commutativity of $\{\hat{E}_{\theta k}\}_{k=0}^{N-1}$ (i.e., $\{\hat{A}_{\theta k}\}_{k=0}^{N-1}$), we can define an observable $(X, \mathscr{F}, F) = (\mathbf{R}^N, \mathscr{B}^N, F)$ in V such that

$$F(\Xi_0 \times \Xi_1 \times \cdots \times \Xi_{N-1}) = \prod_{k=0}^{N-1} \hat{E}_{\theta k}(\Xi_k)$$

Put $v(x_1, \ldots, x_{N-1}) = v_1(x_1) \cdots v_{N-1}(x_{N-1}) \in L^2(\mathbf{R}^{N-1})$ $(\equiv K)$, where $v(x_k)$ is defined by, for $k = 1, \ldots, N-1$,

$$v_k(x_k) = \left(\frac{\omega_k}{\pi}\right)^{1/4} \exp\left(-\frac{\omega_k |x_k|^2}{2}\right) \qquad [\omega_k(>0) \text{ is decided afterward}]. \quad (8)$$

It is easy to show that $\|v_k\|_{L^2(\mathbf{R})} = 1$ (i.e., $\|v\|_K = 1$) and $\int_{\mathbf{R}} x_k |v_k(x_k)|^2\, dx_k = 0$. Put $f_k : X$ $(\equiv \mathbf{R}^N) \to \mathbf{R}$ ($k = 0, 1, \ldots, N-1$) such that $f_k(x_0, \ldots, x_{N-1}) = x_k$. Note that $\hat{A}_{\theta k} = \int_X f_k(x)\, F(dx)$.

It was shown in Ishikawa (1991*b*) that $\mathbf{M} = (K, v, (X, \mathscr{F}, F), f = (f_0, \ldots, f_{N-1}))$ defined above is an approximate simultaneous measurement of $\{A_{\theta k}\}_{k=0}^{N-1}$ in H. Moreover, from the properties of the observables $\{\hat{A}_{\theta k}\}_{k=0}^{N-1}$ and the state $u \otimes v$, we can easily show the following equality ($k = 0, 1, \ldots, N-1$):

$$\mathscr{F}^{u\otimes v}_{A_{\theta k}\otimes I} = \left\{\prod_{k=0}^{N-1} \Xi_k \in \mathscr{B}^N \middle| \hat{E}_{\theta n}(\Xi_n) = 0 \text{ or } I, \text{ if } n \neq k\right\} \quad (9)$$

Therefore, $\mu_{u\otimes v}(x, d\xi)$ in Axiom 2 is given by, for $k = 0, 1, \ldots, N-1$,

$$\mu_{u\otimes v}(x, d\xi) = \lim_{\Xi_k \to \{x_k\}} \frac{\langle u \otimes v, \hat{E}_{\theta k}(\Xi_k) E_{A_{\theta k}\otimes I}(d\xi) u \otimes v\rangle}{\langle u \otimes v, \hat{E}_{\theta k}(\Xi_k) u \otimes v\rangle} \quad (10)$$

where $x = (x_0, \ldots, x_{N-1}) \in \mathbf{R}^N$.

Note that the probability that the measurement value $x = (x_0, \ldots, x_{N-1})$ obtained by the measurement $\mathbf{M}$ belongs to a set $\Xi_0 \times \cdots \times \Xi_{N-1}$ is given by

$$\left\langle u \otimes v, \prod_{k=0}^{N-1} \hat{E}_{\theta k}(\Xi_k)(u \otimes v)\right\rangle \quad (11)$$

Also, when we obtain the measurement value $x = (x_0, \ldots, x_{N-1})$ by the measurement $\mathbf{M}$, it is considered that the expectation $\bar{x} = (\bar{x}_0, \ldots, \bar{x}_{N-1})$ of the "true" value $\hat{x} = (\hat{x}_0, \ldots, \hat{x}_{N-1})$ of the observable $A_{\theta k} \otimes I$ ($k = 0, 1, \ldots, N-1$) for the state $u \otimes v$ is given by

$$\begin{aligned} \bar{x}_k(x) &\equiv \int_{\mathbf{R}} \xi \mu_{u\otimes v}(x, d\xi; (X, \mathscr{F}^{u\otimes v}_{A_{\theta k}\otimes I}, F), A_{\theta k} \otimes I) \\ &= \lim_{\Xi_k \to \{x_k\}} \int_{\mathbf{R}} \xi \frac{\langle u \otimes v, \hat{E}_{\theta k}(\Xi_k) E_{A_{\theta k}\otimes I}(d\xi) u \otimes v\rangle}{\langle u \otimes v, \hat{E}_{\theta k}(\Xi_k) u \otimes v\rangle} \end{aligned} \quad (12)$$

Note that (2) and (10)–(12) can be numerically calculated. So we can know the discrete trajectory of the quantum particle P. Of course, this trajectory can be considered to be a reasonable one that can be inferred from the results of the measurement $\mathbf{M}$.

3. NUMERICAL RESULTS

In this section, we numerically analyze the trajectories of a particle in a two-slit experiment under the theory constructed in the previous section, and give some numerical results. For simplicity, we suppose that $\hbar/m = 1$, and we fix $T = \theta(N-1) = 1/4$.

First we numerically calculate the time evolution of the state $u(t,x) = \exp(-i\hbar^{-1}\mathscr{H}t)u(x)$ $(-\infty < t < \infty)$. The graphs of $|u(0,x)|^2$, $|u(T/2,x)|^2$, and $|u(T,x)|^2$ are presented in Fig. 1. We investigate the trajectories of a particle in the cases that $N=2$ and $N=3$.

3.1. The Case That $N = 2$ (i.e., $\theta = 1/4$)

For this case we easily get (also see Ishikawa (1991*b*))

$$\bar{\delta}_{\mathbf{M}}(A_0, u) = |\alpha_{01}|\left[\int_{\mathbf{R}} |x_1 v_1(x_1)|^2\, dx_1\right]^{1/2}$$
$$= |\alpha_{01}|(2\omega_1)^{-1/2} = \frac{|\alpha_{01}|}{(2\omega_1)^{1/2}}$$
$$\bar{\delta}_{\mathbf{M}}(A_\theta, u) = |\alpha_{11}|\left[\int_{\mathbf{R}} \left|\left(x_1 - i\theta\frac{\partial}{\partial x_1}\right)v_1(x_1)\right|^2 dx_1\right]^{1/2}$$
$$= |\alpha_{11}|\left(\frac{1}{2\omega_1} + \frac{\omega_1|\theta|^2}{2}\right)^{1/2}$$

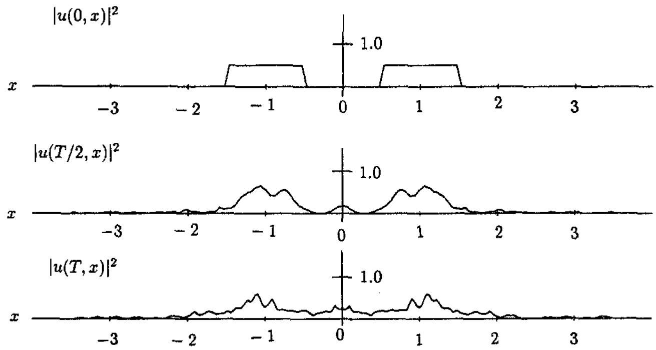

Fig. 1. The time evolution of the state $u(t,x) = \exp(-i\hbar^{-1}\mathscr{H}t)u(x)$.

Note that $\alpha_{01} \cdot \alpha_{11} = -1$, since $\alpha_{00} = \alpha_{10} = 1$. Then we can set $\alpha_{00} = 1$, $\alpha_{10} = 1$, $\alpha_{01} = 2$, and $\alpha_{11} = -1/2$.

For $\omega_1 = 4$, 16, and 64, the values of $\bar{\delta}_{\mathbf{M}}(A_{k\theta}, u)$ $(k = 0, 1)$ are shown in Table I.

Note that the Heisenberg uncertainty relation (3) holds in all cases, that is, $\bar{\delta}_{\mathbf{M}}(A_0, u) \cdot \bar{\delta}_{\mathbf{M}}(A_\theta, u) \geq 1/8$.

We put the intervals in $\mathbf{R}$ at each time $t = k\theta$ $(k = 0, 1, \ldots, N-1)$,

$$\Xi^i_{k\theta} = [-3 + (i-1)/4, -3 + (i/4)), \qquad (i = 1, 2, \ldots, 24)$$

For $i, j = 1, 2, \ldots, 24$ and $k = 0, 1$, we numerically compute the probability that the measurement value $x = (x_0, x_1)$ obtained by the measurement $\mathbf{M}$ belongs to a set $\Xi^i_0 \times \Xi^j_\theta$. These probabilities can be computed from (11). And we can calculate the expectations $\bar{x}_k(x)$ of the "true" value $\hat{x}_k(x)$ from (12).

Figure 2 shows the numerical results of (11) for each $\omega_1 = 4$, 16, and 64. In Fig. 2, we connect Ξ^i_0 and Ξ^j_θ by $[r + (1/2)]$ lines, where $[\cdot]$ is the Gauss symbol and

$$r = 100\langle u \otimes v, \hat{E}_0(\Xi^i_0)\hat{E}_\theta(\Xi^j_\theta)(u \otimes v)\rangle \tag{13}$$

Figure 2 should be viewed with the following considerations:

(a) The average error between the measurement value $x_k(x)$ and "true" value $\hat{x}_k(x)$ is $\bar{\delta}_{\mathbf{M}}(A_{k\theta}, u)$ $(k = 0, 1)$. Therefore, the measurement value is sometimes outside of the two slits at $t = 0$.

(b) The "true" value is produced by measurement $\mathbf{M}$. So this is not the true value in the classical sense.

Figure 3 shows the numerical results of (12) for $\omega_1 = 4$, 16, and 64. Whenever $[r + (1/2)] \geq 1$, we draw a line between $\bar{x}_0(x)$ at $t = 0$ and $\bar{x}_1(x)$ at $t = T$ in Fig. 2.

It is no wonder that sometimes we have $\bar{x}_0(x)$ outside of the slits, since $\bar{x}_0(x)$ is not the "true" value $\hat{x}_0(x)$, but its expectation.

Table I. The Values of Average Error $\{\bar{\delta}_{\mathbf{M}}(A_{k\theta}, u)\}_{k=0}^{1}$ for $\omega_1 > 0$

ω_1	$\bar{\delta}_{\mathbf{M}}(A_0, u)$	$\bar{\delta}_{\mathbf{M}}(A_\theta, u)$
4	0.707 ...	0.250 ...
16	0.353 ...	0.364 ...
64	0.176 ...	0.708 ...

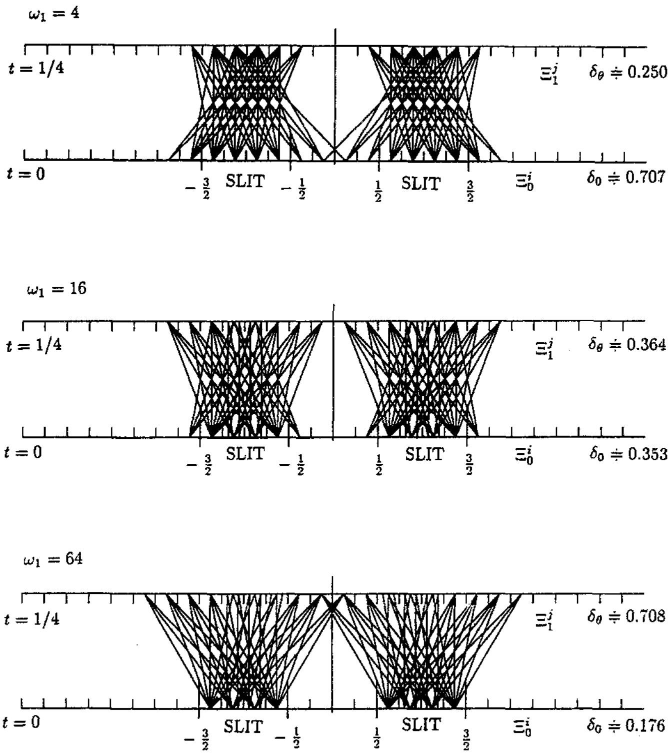

Fig. 2. The representation of the joint distributions of the measurement values at $t = 0$ and $t = T$ for various ω_1.

3.2. The Case That $N = 3$ (i.e., $\theta = 1/8$)

Now we set $\alpha_{01} = -\alpha_{11} = 1.35$, $\alpha_{02} = \alpha_{12} = -0.909$, $\alpha_{21} = 0$, $\alpha_{22} = 1.1$, and $\omega_1 = \omega_2 = 5$. Then

$$\bar{\delta}_{\mathbf{M}}(A_0, u) = \left(\frac{|\alpha_{01}|^2}{2\omega_1} + \frac{|\alpha_{02}|^2}{2\omega_2}\right)^{1/2} = 0.514\ldots$$

$$\bar{\delta}_{\mathbf{M}}(A_\theta, u) = \left[|\alpha_{11}|^2\left(\frac{1}{2\omega_1} + \frac{\omega_1|\theta|^2}{2}\right) + |\alpha_{12}|^2\left(\frac{1}{2\omega_2} + \frac{\omega_2|\theta|^2}{2}\right)\right]^{1/2} = 0.606\ldots$$

$$\bar{\delta}_{\mathbf{M}}(A_{2\theta}, u) = \left[|\alpha_{21}|^2\left(\frac{1}{2\omega_1} + \frac{\omega_1|2\theta|^2}{2}\right) + |\alpha_{22}|^2\left(\frac{1}{2\omega_2} + \frac{\omega_2|2\theta|^2}{2}\right)\right]^{1/2} = 0.556\ldots$$

For $i, j, h = 1, 2, \ldots, 24$ and $k = 0, 1, 2$, we similarly calculate the probability (11) that the measurement value $x = (x_0, x_1, x_2)$ obtained by the measurement $\mathbf{M}$ belongs to a set $\Xi_0^i \times \Xi_\theta^j \times \Xi_{2\theta}^b$ and the expectations $\bar{x}_k(x)$

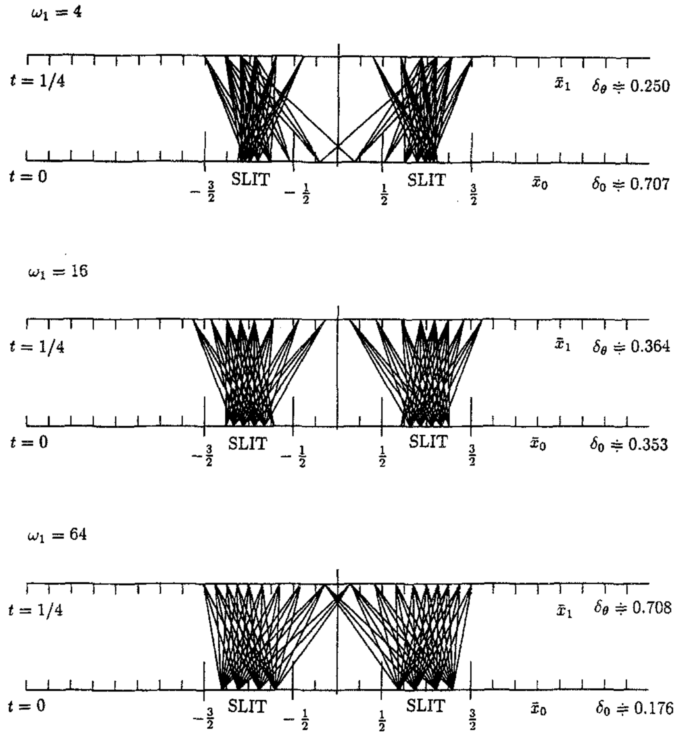

Fig. 3. Plots of the lines among the expectation values $\bar{x}_k$ of "true" values $\hat{x}_k$ for time $t = 0$ and $t = T$ (i.e., $k = 0, 1$).

of the "true" value $\hat{x}_k(x)$ from (12). We shall simulate the experimental measurement of a particle's position as if by rolling a die which has the probability in (11). Figure 4 shows these simulations repeated five times. The obtained trajectories are numbered from 1 to 5. According to the expectation values $\bar{x}_0$ at $t = 0$, $\bar{x}_1$ at $t = T/2$, and $\bar{x}_2$ at $t = T$ of the "true"

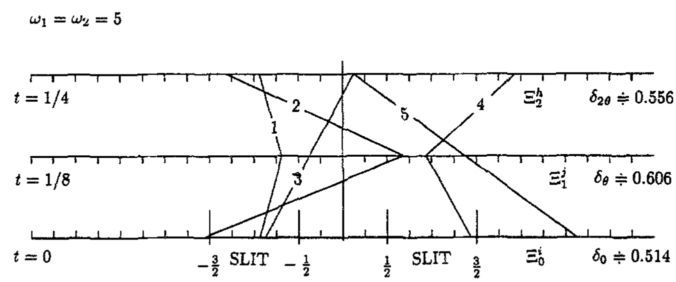

Fig. 4. Plots of the five trajectories obtained by three-point measurements for a particle's position.

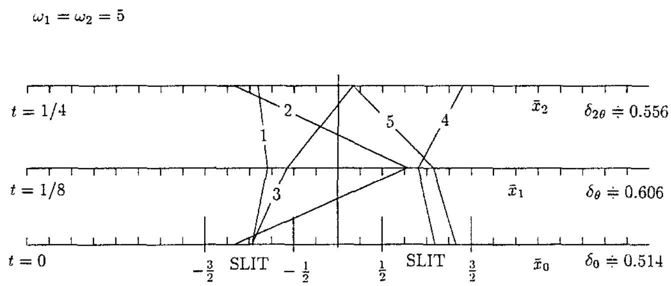

Fig. 5. The modified trajectories as the expectation values of "true" values.

values, we connect the three points $\{(\bar{x}_k, t = k\theta)\}_{k=0}^{2}$ by a line in Fig. 5. The obtained lines are numbered corresponding to the trajectories in Fig. 4.

4. CONCLUSIONS

In this paper we have numerically analyzed discrete trajectories of a quantum particle in two-slit experiments under the interpretation proposed by Ishikawa (1991*b*, 1992). The numerical results obtained in this paper can be considered to be natural and show some justification of this proposal. Of course, we think that more arguments are needed in order to justify this interpretation.

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